THE GENERAL THEORY OF STOCHASTIC POPULATION PROCESSES

BY

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1. Introduction

The purpose of the present paper is to lay down the foundations of a general theory of stochastic population processes (see Bartlett [2] for references to previous work on this subject). By *population* we mean here a collection of individuals, each of which may be found in any one state x of a fixed set of states X. The simplest type of population process is one where there is only one kind of individual and where the total size of the population is always finite with probability unity (finite univariate population process). The state of the whole population is characterized by the states, say $x_1, \ldots, x_n$, of its members, where each of the x_i ranges over X and $n = 0, 1, 2, \ldots$; thus we may have for example a biological population whose individuals are characterized by their age, weight, location, etc. or a population of stars characterized by their brightness, mass, position, velocity and so on. Such a population is *stochastic* in the sense that there is defined a probability distribution P on some σ-field $\boldsymbol{B}$ of subsets of the space $\mathfrak{X}$ of all population states; in § 2 we develop the theory of such *population probability spaces*; the approach is similar to that of Bhabha [4], who, however, restricts himself to the case where X is the real line and P is absolutely continuous. By taking the individual state space X to be arbitrary (i.e., an abstract space), we are able to make the theory completely general, including for example the case of cluster processes, where the members of a given population are clusters, i.e., are themselves populations, as in Neyman's theory of populations of galaxy clusters (cf. Neyman

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