ON THE FOUR ROTATIONS WHICH DISPLACE ONE ORTHOGONAL SYSTEM OF AXES INTO ANOTHER

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This question has been treated by Herr LIPSCHITZ in a very elegant analytical manner in the first three sections of a memoir in the Acta mathematica (Vol. 24, p. 123). It is the object of the following short note to shew that the question is susceptible of a simple kinematical treatment which brings out, even more clearly perhaps than an algebraical process, the essential space-relations of the configuration involved.

The one kinematical theorem of which repeated use is made, — namely that successive rotations about three radii of a sphere through twice the angles of the corresponding spherical triangle produce no displacement at all — is due originally, I believe, to HAMILTON (*Lectures on Quaternions*, p. 267). It is equivalent to a construction for the resultant of any two rotations about intersecting axes. Moreover from the theorem itself the converse — if successive rotations about two radii OP, OQ of a sphere have OR for the axis of their resultant, then the amplitudes of the two rotations are $2RPQ$ and $2PQR$ (mod 2π) — immediately follows. It may be noted that HAMILTON's theorem is established almost intuitively by drawing Op, Oq, Or perpendicular to the planes QOR, ROP, POQ. For successive rotations through two right angles about Oq and Or is the same as a rotation round OP through $2RPQ$. Hence successive rotations through $2RPQ$, $2PQR$, $2QRP$ about OP, OQ, OR are equivalent to successive rotations through two right angles about Oq, Or, Or, Op, Op, Oq, which clearly give no displacement.