# ON THE RELATION OF THE ABELIAN TO THE JACOBIAN ELLIPTIC FUNCTIONS

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1. The three Jacobian elliptic functions $\operatorname{sn} x$, $\operatorname{cn} x$, $\operatorname{dn} x$, their reciprocals and their quotients form a system of twelve functions which I have found it convenient to represent by a uniform notation viz. putting $\frac{\operatorname{sn} x}{\operatorname{cn} x} = \operatorname{sc} x$, $\frac{\operatorname{cn} x}{\operatorname{dn} x} = \operatorname{cd} x$, $\frac{1}{\operatorname{sn} x} = \operatorname{ns} x$, &c, we have twelve functions, each denoted by a functional sign consisting of two of the letters s, c, d, n, and forming the following four groups, the members of each group having the same final letter:

$$\operatorname{sn} x, \operatorname{cn} x, \operatorname{dn} x\ ;\ \operatorname{cd} x, \operatorname{sd} x, \operatorname{nd} x\ ;\ \operatorname{dc} x, \operatorname{nc} x, \operatorname{sc} x\ ;\ \operatorname{ns} x, \operatorname{ds} x, \operatorname{cs} x.$$

Each of these four groups might have been selected as the standard group, the members of the other groups being derived from it merely as reciprocals and quotients. The actual selection of the first group by JACOBI was due to the fact that $(1 - x^2)(1 - c^2x^2)$ was LEGENDRE's standard form of the general quartic function, this form having been chosen by him in order that the denominator of the integral might be reducible to the form $\sqrt{1 - c^2 \sin^2 \varphi}$.

2. In treating the Jacobian theory in my lectures on Elliptic Functions, I have been accustomed for many years to employ the twelve elliptic functions sn, cn, cd, dc, &c. and the four corresponding Zeta functions denoted, by a somewhat analogous notation, by zn, zd, zc, zs. These