SPECTRAL THEORY OF CLOSED DISTRIBUTIVE OPERATORS.

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1. **Introduction.** Let X be a complex Banach space, and T a closed distributive operator with domain and range both in X. Let $[X]$ denote the set of all continuous distributive (bounded linear) operators which map X into itself. This set $[X]$ is a ring, and in fact an algebra. Next suppose that we have an algebra, related in some way to T, whose elements are complexvalued functions of the complex variable λ, and that we are able to define a mapping of the algebra of functions into the algebra $[X]$ in such a way that we have a homomorphism. If $f(\lambda)$ is the complex function, we shall denote the corresponding member of $[X]$ by $f(T)$. The fact that we have a homomorphism is then expressed by the equations

$$(1.1)\qquad \begin{aligned}(af+bg)(T) &= af(T)+bg(T) ,\\ (fg)(T) &= f(T)g(T) .\end{aligned}$$

When such a homomorphism has been established we shall speak of the application of formulas (1.1) and other related results flowing out of the homomorphism as an operational calculus for T.

Some years ago (Dunford, [1 and 2]; Taylor [2])[1] an operational calculus was developed for bounded operators T by choosing as the algebra of functions the set of functions $f(\lambda)$, each singlevalued and analytic in some open set containing the spectrum $\sigma(T)$ of T. The homomorphism was established by defining

$$(1.2)\qquad f(T) = \frac{1}{2\pi i}\int f(\lambda)(\lambda I-T)^{-1}d\lambda ,$$

the integral being extended over the boundary of a suitable bounded domain containing $\sigma(T)$. Dunford [1] used the resulting operational calculus to develop syste-

[1] All references are to the bibliography at the end of the paper.