EUCLID'S ALGORITHM IN CUBIC FIELDS OF NEGATIVE DISCRIMINANT.

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1. Introduction.

Let K be any algebraic number field. If, for each number λ of the field K, there is an algebraic integer ξ of K such that

$$|N(\xi-\lambda)| < 1 ,$$

where N denotes the norm, then Euclid's algorithm is said to be valid in K. For complex quadratic fields, the question is almost trivial. For real quadratic fields, it has been known for some years that there are only a finite number of cases in which Euclid's algorithm is valid. I have recently given[1] a proof of this result based on new principles, and this proof has led to the complete enumeration[2] of all such cases.

Now let K be a cubic field of negative discriminant, that is, a field generated by a real cubic irrationality whose conjugates are complex. The main result of the present paper is that *Euclid's algorithm is valid only in a finite number of such fields.*

As in the quadratic case, the result is closely connected with one which relates to a more general situation. Let

$$(1) \qquad \begin{cases} \xi = \alpha u+\beta v+\gamma w , \\ \xi' = \alpha' u+\beta' v+\gamma' w , \\ \xi''= \alpha'' u+\beta'' v+\gamma'' w \end{cases}$$

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[1] "Indefinite binary quadratic forms, and Euclid's algorithm in real quadratic fields", *Proc. London Math. Soc.* (in course of publication).

[2] See H. Chatland and H. Davenport, "Euclid's algorithm in real quadratic fields", *Canadian J. of Math.* (in course of publication).