A CLASS OF SPECIAL $\mathcal{L}_\infty$ SPACES

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1. Introduction

We shall construct Banach spaces X and Y having some peculiar properties.

(a) X is a separable $\mathcal{L}_\infty$ space.

(b) X is a Radon-Nikodym space. Since a separable $\mathcal{L}_\infty$ space cannot be imbedded isomorphically into a separable dual space, this example solves negatively the following conjecture of Uhl: Is every separable Radon-Nikodym space isomorphic to a subspace of a separable dual space?

(c) X is a Schur space, i.e. weak and norm compactness coincide in X. This answers negatively a conjecture of Lindenstrauss who asked in [10] whether a space which has the weakly compact extension property is necessarily finite dimensional (see also Theorem 2.4). In [11] Pełczynski and Lindenstrauss and in [12] Lindenstrauss and Rosenthal asked whether every $\mathcal{L}_\infty$ space contains a subspace isomorphic to c_0. Our example disproves this conjecture.

(d) X is weakly sequentially complete. Since X^* is a $\mathcal{L}_1$ space, X^* is also weakly sequentially complete. For a long time it was conjectured that a Banach space is reflexive if and only if both X and X^* are weakly sequentially complete.

(a′) The Banach space Y is a separable $\mathcal{L}_\infty$ space.

(b′) The Banach space Y is a Radon-Nikodym space.

(c′) Y is somewhat reflexive, i.e. every infinite dimensional subspace of Y contains an infinite dimensional subspace which is reflexive.

Since Y does not contain a subspace isomorphic to l_1 it follows from results of Lewis and Stegall [9] that Y^* is isomorphic to l_1. It is strange that Y does not contain a copy of c_0. Since Y is a $\mathcal{L}_\infty$ space it also has the Dunford-Pettis property and hence there are