DEFORMATION CLASSES OF MEROMORPHIC FUNCTIONS AND THEIR EXTENSIONS TO INTERIOR TRANSFORMATIONS.

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§ 1. *Introduction.* The first objective of this paper is to use topological methods and concepts to enlarge the store of knowledge of meromorphic functions. Deformation classes of meromorphic functions are defined. The extension to interior transformations results in new homotopy theorems and contrasts between interior and conformal maps. Earlier papers have shown that there is a considerable body of theorems which can be formulated so as to retain meaning and validity after arbitrary homeomorphisms of the z- or w-spheres. Many theorems involving the relations between zeros, poles, and branch point antecedents and the images under f of boundaries are of this character. See Morse and Heins, (1) and Morse (2).

The second objective is to distinguish between the properties of meromorphic functions which are shared by interior transformations and those which are not. For the transformations from $\{|z|<1\}$ to the w-sphere which are considered we find no difference[1] between meromorphic functions and interior transformations with respect to the invariants necessary to characterize a deformation class of functions with prescribed zeros, poles, and branch point antecedents. However, sequences $[f_k(z)]$ of meromorphic functions properly taken from different deformation classes cover the w-plane in a manner suggestive of the Picard theorem on essential singularities but with no counterpart for sequences of interior transformations. The discovery of such properties points to the problem of finding the non-topological assumptions which must be imposed upon interior transformations in order that they may share the non-topological properties discovered.

[1] For domains for z other than the disc differences may arise.