ON THE THEORY OF INTERPOLATION.

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Introduction.

Let us denote

(1) $$x_1^{(n)},\ x_2^{(n)},\ \ldots,\ x_n^{(n)}$$

n distinct points in the interval $-1 \leqq x \leqq +1$ and let $f(x)$ be a function defined in the same interval. We investigate in this note the convergence problems of the Lagrange and Hermite interpolation polynomials of the function $f(x)$ corresponding to the "fundamental points" (1). The n^{th} Lagrange interpolation polynomial of $f(x)$ is the unique polynomial of degree $n-1$ at most, assuming the values $f(x_1^{(n)})$, $f(x_2^{(n)})$, ..., $f(x_n^{(n)})$ at the abscissas $x_1^{(n)}$, $x_2^{(n)}$, ..., $x_n^{(n)}$ respectively. This polynomial is given by the formula

(2) $$L_n[f] = \sum_{k=1}^{n} f(x_k^{(n)})\, l_k^{(n)}(x);$$

here

(3) $$l_k^{(n)}(x) = \frac{\omega_n(x)}{\omega_n'(x_k^{(n)})\,(x - x_k^{(n)})}$$

and the polynomial $\omega(x)$ defined by

(4) $$\omega(x) = (x - x_1^{(n)})(x - x_2^{(n)}) \ldots (x - x_n^{(n)}).$$

The n^{th} Hermite interpolation polynomial of $f(x)$ is the unique polynomial of degree at most $2n-1$ which for the values $x_1^{(n)}$, $x_2^{(n)}$, ..., $x_n^{(n)}$ assumes, respectively, the values $f(x_1^{(n)})$, $f(x_2^{(n)})$, ..., $f(x_n^{(n)})$ and whose derivative correspondingly assumes the given values $d_1^{(n)}$, $d_2^{(n)}$, ..., $d_n^{(n)}$. The explicite form of this polynomial is given by the formula