# NEW INTEGRALS INVOLVING BESSEL FUNCTIONS

BY

F. M. RAGAB

*Ein Shams University, Abbassia, Cairo*

The formula [T. M. MacRobert, Functions of a Complex Variable, 4th ed., Glasgow 1954, p. 406, (1)] namely

$$K_m(x)\,K_n(x)=\frac{\sqrt{\pi}}{4\,\pi\,x}\sum_{i,\,-i}\frac{1}{i}\,E\left(\frac{1+m+n}{2},\,\frac{1+m-n}{2},\,\frac{1+n-m}{2},\,\frac{1-n-m}{2}:\frac{1}{2}:e^{i\pi}x^2\right),\quad (1)$$

will be used to evaluate a large number of integrals involving Bessel Functions which are all entirely new.

Thus from (1), *loc. cit.*, and formula (1)

$$\int_0^\infty e^{-\lambda}\lambda^{k-1}E\left(p;\,\alpha_r:q;\,\varrho_s:\frac{x}{\lambda^n}\right)d\lambda=(2\pi)^{\frac{1}{2}-\frac{1}{2}n}\cdot n^{k-\frac{1}{2}}\,E\left(p+n;\,\alpha_r:q;\,\varrho_s:\frac{x}{n^n}\right),\quad (2)$$

where $\qquad R(k)>0,\ \alpha_{p+\nu+1}=\dfrac{k+\nu}{n},\qquad \nu=0,\,1,\,2,\,\ldots,\,n-1,$

one gets, if $x$ is real and positive,

$$\int_0^\infty e^{-\lambda}\lambda^{k-1}K_m\left(\frac{x}{\lambda}\right)K_n\left(\frac{x}{\lambda}\right)d\lambda$$

$$=\frac{2^{k-2}}{\pi x}\sum_{i,\,-i}\frac{1}{i}\,E\left(\frac{1+m+n}{2},\,\frac{1+m-n}{2},\,\frac{1+n-m}{2},\,\frac{1-n-m}{2},\,\frac{k+1}{2},\,\frac{k+2}{2}:\frac{1}{2}:\frac{e^{i\pi}x^2}{4}\right).\qquad (3)$$

Also from (1) and formula (2)

$$\int_0^\infty e^{-\lambda}\lambda^{k-1}E(p;\alpha_r:q;\varrho_s:\lambda^m x)\,d\lambda=\pi\,\operatorname{cosec}(k\pi)\,(2\pi)^{\frac{1}{2}m-\frac{1}{2}}\,m^{k-\frac{1}{2}}\times$$

$$\times E\left(p;\,\alpha_r:1-\frac{k}{m},\,1-\frac{k+1}{m},\,\ldots,\,1-\frac{k+m-1}{m},\,\varrho_1,\,\ldots,\,\varrho_q:e^{\pm m\pi i}m^m z)\right)+$$