THE MEAN-VALUE OF THE RIEMANN ZETA FUNCTION.

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1. It was shown by Hardy and Littlewood[1] that an analogue of the ordinary mean-value theorem for Dirichlet series exists for the Riemann zeta function on the critical line. Writing

$$I(T)=\int_0^T |\zeta(\tfrac{1}{2}+it)|^2\,dt$$

they showed that

$$I(T)\sim T\log T \tag{1.1}$$

as $T\to\infty$.

A substantial advance was made by Littlewood[2] who proved that

$$I(T)=T\log T-T(1+\log 2\pi-2\gamma)+O(T^{\frac{3}{4}+\epsilon}) \tag{1.2}$$

by means of results connected with the approximate functional equation for $\zeta(s)$. Using improved forms of this equation Ingham[3] and Titchmarsh[4] were able to reduce the power of T in the error term in (1.2) to $\frac{1}{2}$ and $^5/_{12}$ respectively.

The problem has more than superficial affinities with a well-known divisor problem[5], namely the behaviour of

$$\sum_{n<x} d(n). \tag{1.3}$$

Here (1.2) is the analogue of Dirichlet's classical formula and its later refinements. My object in this paper is to establish what corresponds, in the case

[1] G. H. HARDY and J. E. LITTLEWOOD, Acta math. 41 (1918), 119—196.

[2] J. E. LITTLEWOOD, Proc. London Math. Soc. (2), 20 (1922), Records, XXII—XXVIII.

[3] A. E. INGHAM, Proc. London Math. Soc. (2), 27 (1926), 273—300.

[4] E. C. TITCHMARSH, Quart. J. of Math. (Oxford), 5 (1934), 195—210.

[5] The connection is to some extent apparent in my paper, Quart. J. of Math. (Oxford), 10 (1939), 122—128. I hope to go in this question more deeply in a subsequent paper.

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