ON LATTICE POINTS IN A CONVEX DECAGON.

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Let K be a convex domain in the (x, y)-plane symmetrical in the origin $O = (0, 0)$ of the coordinate system. If

$$X_1 = (x_1, y_1) \text{ and } X_2 = (x_2, y_2)$$

are two points not collinear with O, then the set Λ of all points[1]

$$u_1 X_1 + u_2 X_2 \qquad (u_1, u_2 = 0, \mp 1, \mp 2, \ldots)$$

is a lattice, and the positive number

$$d(\Lambda) = |(X_1, X_2)|$$

is the determinant of Λ. We say that Λ is *K-admissible* if no point of Λ except O is an *inner* point of K. Then the lower bound

$$\Delta(K) = \text{l. b. } d(\Lambda)$$

extended over all K-admissible lattices is a positive number and is called the *minimum determinant of K*. There exist critical lattices of K, i. e. lattices Λ which are K-admissible and of determinant

$$d(\Lambda) = \Delta(K).$$

Except when K is a parallelogram, such lattices have just three pairs of points $\mp A$, $\mp B$, $\mp C$ on the boundary of K, and if the notation is chosen suitably, then

$$A + B = C.$$

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[1] We use vector notation; thus $u_1 X_1 + u_2 X_2 = (u_1 x_1 + u_2 x_2,\ u_1 y_1 + u_2 y_2)$, and in particular $-X_1 = (-x_1, -y_1)$. The determinant of X_1 and X_2 is denoted by $(X_1, X_2) = x_1 y_2 - x_2 y_1$.