SOME THEOREMS ON ALGEBRAIC RINGS.

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In his paper "Sätze über algebraische Ringe"[1] T. Nagell has discussed certain properties of algebraic rings. The present note concerns itself with the generalization of these results to relative algebraic rings; the theorems will be transferred without essential change.

In what follows we shall mean by F a finite algebraic number field and by R the ring of the integral elements of F. Let further ϕ be an algebraic field over F of degree n and let P be the ring of the integral elements of ϕ. It is well known that in ϕ there are n elements[2], $\omega_1, \ldots, \omega_n$, being linearly independent with respect to F, such that every element of ϕ possesses a unique representation of the form

$$\omega = a_1 \omega_1 + \cdots + a_n \omega_n \tag{1}$$

with coefficients in F. The ω_i are called the basis of ϕ with respect to F. Let ξ be an element of P of the exact degree n, that is, ξ is a root of an *irreducible* algebraic equation $x^n + r_1 x^{n-1} + \cdots + r_n = 0$ where r_i are in R. In view of (1) we may set

$$\xi^k = c_{k1} \omega_1 + \cdots + c_{kn} \omega_n, \qquad (c_{ki} \varepsilon F) \tag{2}$$

for $k = 0, 1, \ldots, n-1$. Since ξ was chosen so as to be of the exact degree n, the determinant $c = |c_{ki}|$ of the coefficients in (2) does not vanish, and so the system may be inverted, and then we get

$$\omega_i = \frac{1}{c}(b_{i1} + b_{i2}\xi + \cdots + b_{in}\xi^{n-1}), \qquad (b_{ik} \varepsilon F) \tag{3}$$

for $i = 1, 2, \ldots, n$.

[1] Math. Zeitschrift 34 (1932), pp. 179—182.

[2] The elements of F will be denoted by Latin, those of ϕ by Greek letters.