LACUNAS FOR HYPERBOLIC DIFFERENTIAL OPERATORS WITH CONSTANT COEFFICIENTS I

BY

M. F. ATIYAH, R. BOTT, L. GÅRDING

Introduction

Ивану Георгиевичу Петровскому
с уважением и восхищением

The theory of lacunas for hyperbolic differential operators was created by I. G. Petrovsky who published the basic paper of the subject in 1945 (1). Although its results are very clear, the paper is difficult reading and has so far not lead to studies of the same scope. We shall clarify and generalize Petrovsky's theory.

The various kinds of linear free wave propagation that occur in the mathematical models of classical physics are of the following general type. There is an elastic $(n-1)$-dimensional medium whose deviation from rest position is described by a function $u(x)$ with values in some R^N and defined in some open subset O of R^n, one of the coordinates being time. When there are no exterior forces, u satisfies a system of N linear partial differential equations $P(x, \partial/\partial x)u(x)=0$ where P is a linear partial differential operator with smooth matrix-valued coefficients. Further, a unit impulse applied at some point y produces close to y a movement that propagates with a locally bounded velocity in all directions. This movement is smooth outside a system of possibly criss-crossing wave fronts and vanishes outside the cone of propagation $K(P, y)$, a conical region with its vertex at y and bounded by the fastest fronts. Mathematically, the movement is described by a distribution $E=E(P, x, y)$ which is defined when x is close to y, vanishes when x is outside $K(P, y)$ and satisfies

$$P(x, \partial/\partial x)\, E(P, x, y) = \delta(x-y)$$

so that E is a (right) fundamental solution of P. Under these circumstances we say that P is a hyperbolic operator. Briefly, P is hyperbolic if it has a fundamental solution with

(1) See the references.