

# SOME PROPERTIES OF TRIGONOMETRIC SERIES WHOSE TERMS HAVE RANDOM SIGNS

*Dedicated to Professor Hugo Steinhaus for his 65th Birthday*

BY

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Trigonometric series of the type

$$(0.1) \quad \sum_1^{\infty} \varphi_n(t) (a_n \cos nx + b_n \sin nx),$$

where  $\{\varphi_n(t)\}$  denotes the system of Rademacher functions, have been extensively studied in order to discover properties which belong to "almost all" series, that is to say which are true for almost all values of  $t$ .<sup>1</sup> We propose here to add some new contributions to the theory.

## CHAPTER I

### Weighted Means of Ortho-normal Functions

1. Let  $\varphi_1, \varphi_2, \dots, \varphi_n, \dots$  be a system of functions of  $x$ , ortho-normal in an interval  $(a, b)$ , and let  $\gamma_1, \gamma_2, \dots, \gamma_n, \dots$  be a sequence of non-negative constants such that

$$S_n = \gamma_1 + \gamma_2 + \dots + \gamma_n$$

increases indefinitely as  $n$  tends to  $+\infty$ . Under what conditions does the mean

$$R_n(x) = \frac{\gamma_1 \varphi_1(x) + \gamma_2 \varphi_2(x) + \dots + \gamma_n \varphi_n(x)}{\gamma_1 + \gamma_2 + \dots + \gamma_n}$$

tend to zero almost everywhere<sup>2</sup> as  $n \rightarrow \infty$ ?

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<sup>1</sup> Cf., in particular, PALEY and ZYGMUND, *Proc. Cambridge Phil. Soc.*, 26 (1930), pp. 337–357 and 458–474, and 28 (1932), pp. 190–205.

<sup>2</sup> We write briefly  $R_n(x) \rightarrow 0$  p.p. ("presque partout").