ON THE EXCEPTIONAL GROUP OF A WEIERSTRASS CURVE IN AN ALGEBRAIC FIELD

BY

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1. Let A and B be two numbers satisfying

$$4A^3 - 27B^2 \neq 0. \tag{1}$$

Then

$$y^2 = x^3 - Ax - B \tag{2}$$

is a curve of genus 1, and its coordinates can be represented by

$$\begin{cases} x = \wp(u;\, 4A,\, 4B); \\ y = \tfrac{1}{2}\wp'(u;\, 4A,\, 4B). \end{cases} \tag{3}$$

If ω, ω' is a primitive pair of periods of the $\wp$-function, the number u is determined mod ω, ω' by the point (x, y), which will be called *the point* u. If ν is a rational integer, and if

$$\nu u \not\equiv 0 \pmod{\omega, \omega'},$$

the coordinates of the point νu will generally be denoted by (x_ν, y_ν). Three points u_1, u_2, u_3 lie on a straight line, if

$$u_1 + u_2 + u_3 \equiv 0 \pmod{\omega, \omega'},$$

and conversely. If the number u is commensurable with a period, and if q is the smallest natural number that makes qu a period, then u is called an *exceptional point of order* q (see Nagell [7]). Since there are two independent periods, there exist q^2 exceptional points, whose orders divide q. The point of order 1 is the infinite point of inflexion, and the points of order 2 are given by $y = 0$.

If A and B belong to a field $\boldsymbol{\Omega}$, a point on (2) is said to be a *point in* $\boldsymbol{\Omega}$, if its coordinates belong to this field. If u_1 and u_2 are exceptional points in $\boldsymbol{\Omega}$, the

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