CURVE FAMILIES F^* LOCALLY THE LEVEL CURVES OF A PSEUDOHARMONIC FUNCTION

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Introduction

The family F^* may be defined over an arbitrary open Riemann surface Q. When Q is not simply connected there may exist no single-valued PH [pseudoharmonic] function on Q with F^* as its family of level lines. On the universal covering surface M of Q there do exist PH functions u, single-valued on M and with a family F^*_M of level lines which projects into F^* on Q. While u may not be single-valued on Q it may behave like an integral in that it has branches which differ by a constant, or it may have a real logarithm which has this property. In studying such behavior of u one may focus on the branches of u obtained by continuation of u along a single closed curve k not homotopic to zero on Q.

In this way one is led to the essentially typical case of a family F^* defined on a sphere Σ^* with a north pole N and south pole S removed. Although there may be no single-valued PH function u on Σ^* with F^* as its family of level lines there will in general be multiple-valued functions u satisfying linear relations

$$u[p^{(1)}] = a\,u(p) + b \qquad (a \neq 0) \tag{1.0}$$

where p and $p^{(1)}$ are points on the universal covering surface M of Σ^*, and where p and $p^{(1)}$ in M project into the same point in Σ^*, but on M have longitudes θ and $\theta + 2\pi$ respectively. However the values of the constants a and b for which a relation (1.0) may hold depend in a deep way upon the nature of the family F^*. See MJ 4 and MJ 5.

In the present paper we decompose Σ^* into canonical regions, "primitives," "caps," "annuli," "polar sectors," "cut sectors," etc., whose nature is determined by F^*. With F we associate integral indices $\nu(F)$ and $\mu(F)$ [defined in a later paper]. The existence of PH functions u satisfying prescribed linear relations (1.0) depends upon these indices and upon the character of the decomposition of Σ^*.