

Characters, asymptotics and \mathfrak{n} -homology of Harish-Chandra modules

by

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§ 1. Introduction

Let π be an irreducible representation of a semisimple Lie group G . As has been known for some time, there exist connections between three types of invariants of π : the asymptotic behavior of its “matrix coefficients”, the character of π , and the set of induced representations into which π can be embedded. Most of the analytic arguments in the representation theory of G exploit these connections in some way. Harish-Chandra’s construction of the discrete series, for instance, is based on a detailed analysis of the interaction between the growth rates of the character and of the matrix coefficients. To give a second example, Langlands classifies the irreducible representations π by realizing them as subrepresentations of certain induced representations, which he describes in terms of the asymptotic behavior of the matrix coefficients of π . In this paper, we systematically explore the relationships between characters, asymptotics, and embeddings into induced representations.

Our main tool is a character identity that was conjectured by Osborne [33]. In order to explain the conjecture, we consider a parabolic subgroup $P=LN$, with unipotent radical N and Levi factor L . In the special case of a finite dimensional representation π , the group L operates naturally on the Lie algebra homology groups $H_p(\mathfrak{n}, V)$ of the representation space V , with respect to the complexified Lie algebra \mathfrak{n} of N ; this action is induced by the action of L on the standard complex of Lie algebra homology

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