

THE SPACE OF GROUPOIDS OVER A COMPACTUM*.

By

DAVID ELLIS and GAINES LANG

in Gainesville, Florida, U. S. A.

1. Introduction. Numerous studies have been made concerning the topology of spaces of functions of various types (continuous, measurable, etc.) over various types of topological spaces. These functions generally take values in a linear normed space. It seems also desirable to consider the set of algebraic functions (that is, binary, single-valued operations) defined in a given space. This is the object of the present paper.

We restrict attention to the case when our underlying space is a compactum (= compact metric space). Our definition of distance in the space of groupoids applies to any bounded semimetric space and a similar function may be defined even in the case of an unbounded semimetric space by the introduction of a "convergence factor" such as is used by Busemann¹ in the group of motions of a distance space. However, the restriction to a compactum is used in the present paper to obtain our desired results.

Throughout, we use the axiom of choice without further comment. We also employ as a Lemma in Section 3 the well-known Policeman's theorem:

Lemma 0. *Let M be a compactum and $\epsilon > 0$. There is a finite subset of M which is ϵ -dense in M .*

We consider first some fundamental metric and topological properties of the space of groupoids and then turn to the closure of certain subsets of this space whose elements are of particular interest in topological algebra. The final section discusses the major unsolved problem and a suggestion for related study.

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¹ HERBERT BUSEMANN, *Local metric geometry*, Trans. Am. Math. Soc., vol. 56 (1944), pp. 260