A THEORY OF RADON MEASURES ON LOCALLY COMPACT SPACES.

By

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1. Introduction.

In functional analysis one is often presented with the following situation: a locally compact space X is given, and along with it a certain topological vector space $\mathcal{E}$ of real functions defined on X; it is of importance to know the form of the most general continuous linear functional on $\mathcal{E}$. In many important cases, $\mathcal{E}$ is a superspace of the vector space $\mathfrak{C}$ of all real, continuous functions on X which vanish outside compact subsets of X, and the topology of $\mathcal{E}$ is such that if a sequence (f_n) tends uniformly to zero and the f_n collectively vanish outside a fixed compact subset of X, then (f_n) is convergent to zero in the sense of $\mathcal{E}$. In this case the restriction to $\mathfrak{C}$ of any continuous linear functional μ on $\mathcal{E}$ has the property that $\mu(f_n) \to 0$ whenever the sequence (f_n) converges to zero in the manner just described. It is therefore an important advance to determine all the linear functionals on $\mathfrak{C}$ which are continuous in this sense.

It is customary in some circles (the Bourbaki group, for example) to term such a functional μ on $\mathfrak{C}$ a "Radon measure on X". Any such functional can be written in many ways as the difference of two similar functionals, each having the additional property of being positive in the sense that they assign a number ≥ 0 to any function f satisfying $f(x) \geq 0$ for all $x \in X$. These latter functionals are termed "positive Radon measures on X", and it is to these that we may confine our attention.

It is a well known theorem of F. Riesz (Banach [1], pp. 59—61) that if X is the compact interval $[0, 1]$ of the real axis, then any positive linear functional μ on $\mathfrak{C}$ has a representation in the form

$$\mu(f) = \int_0^1 f(x)\, d\, V(x).$$

$V(x)$ being a certain bounded, non-decreasing point-function on $[0, 1]$. When X is a general locally compact space, the problem has been treated (albeit in a rather incidental