On elliptic systems in $\mathbf{R}^n$

by

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1. Statement of results

This paper studies elliptic $k\times k$ systems of partial differential operators in $\mathbf{R}^n$ which may be written in the form

$$A = A_\infty + Q \tag{1.1}$$

where A_∞ is an elliptic system of constant coefficient operators and Q is a variable coefficient perturbation with certain decay properties at $|x|=\infty$.

For the case $k=1$ such operators were studied in [6], [7] and [8] under the conditions

$$\begin{gathered} A_\infty \text{ is an elliptic constant coefficient} \\ \text{operator which is homogeneous of degree } m \end{gathered} \tag{1.2}$$

and the coefficients of

$$Q = \sum_{|\alpha|\leq m} q_\alpha(x)\,\partial^\alpha$$

satisfy $q_\alpha \in C^l(\mathbf{R}^n)$ and

$$\overline{\lim_{|x|\to\infty}} \left| \langle x \rangle^{m-|\alpha|+|\beta|} \partial^\beta q_\alpha(x) \right| = C_{\alpha,\beta} < \infty \tag{1.3}$$

for all $|\beta|\leq l \in \mathbf{N}$. (Here and throughout this paper we let $\mathbf{Z}$ denote the integers, $\mathbf{N}$ denote the nonnegative integers, $\langle x\rangle = (1+|x|^2)^{1/2}$, $p'=p/(p-1)$, and use standard conventions for multi-indices $\alpha=(\alpha_1,\ldots,\alpha_n)\in\mathbf{N}^n$ and $\partial^\alpha = (\partial/\partial x_1)^{\alpha_1}\ldots(\partial/\partial x_n)^{\alpha_n}$.)

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[1] Research partially supported by NSF Grant MCS-80-02995