

ANALYTIC HYPOELLIPTICITY OF THE $\bar{\partial}$ -NEUMANN PROBLEM AND EXTENDABILITY OF HOLOMORPHIC MAPPINGS

BY

STEVEN R. BELL⁽¹⁾

Princeton University, New Jersey, U.S.A.

1. Introduction

In the theory of functions of one complex variable, the proof of the theorem which states that a proper holomorphic mapping between domains with real analytic boundaries extends holomorphically past the boundary consists of two relatively simple steps: first prove that such mappings extend continuously to the boundary; then apply the classical Schwarz reflection principle. Attempts to generalize these techniques to mappings in several complex variables have not been entirely successful. The principle reasons for this are: (1) there is not a satisfactory reflection principle for weakly pseudoconvex hypersurfaces in \mathbb{C}^n , and (2) proper maps in \mathbb{C}^n may branch at boundary points. In this paper, we attempt to expose the connection between the problem of extending proper holomorphic mappings and the real analytic hypoellipticity of the $\bar{\partial}$ -Neumann problem. To be precise, we prove that if D_1 and D_2 are bounded domains with real analytic boundaries, and if the $\bar{\partial}$ -Neumann problem for D_1 is globally real analytic hypoelliptic, then any proper holomorphic mapping f of D_1 onto D_2 extends holomorphically to a neighborhood of $\overline{D_1}$. This result allows us to prove that there can be no proper holomorphic mapping of a bounded domain with real analytic boundary which is strictly pseudoconvex onto such a domain which is weakly pseudoconvex. When our techniques are localized, we are able to prove that if $f: D_1 \rightarrow D_2$ is a proper holomorphic mapping between bounded pseudoconvex domains with real analytic boundaries, then f maps the set Γ of strictly pseudoconvex boundary points of D_1 into the set of strictly pseudoconvex boundary points of D_2 . Furthermore, f extends holomorphically past Γ and is unbranched on Γ .

It should be pointed out that the general problem of proving the global analytic hypoellipticity of the $\bar{\partial}$ -Neumann problem in a weakly pseudoconvex domain with real

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