ON COMPACT KÄHLER MANIFOLDS OF NONNEGATIVE BISECTIONAL CURVATURE, I

BY

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This is the first of two papers devoted to the study of compact Kähler manifolds of nonnegative bisectional curvature (see also [Wu2]). The main result of this paper is the following theorem; together with its corollaries below, this theorem shows that such manifolds possess a rigid internal structure. For their statements, recall from [Wu1] that the Ricci curvature is *quasi-positive* iff it is everywhere nonnegative and is positive in all directions at a point; an equivalent definition is that the Ricci tensor Ric is everywhere positive semi-definite and is positive definite at a point.

THEOREM. *Let M be an n-dimensional compact Kähler manifold with nonnegative bisectional curvature and let the maximum rank of* Ric *on M be $n-k$ $(0 \leq k \leq n)$. Then:*

(A) *The universal covering of M is holomorphically isometric to a direct product $M' \times \mathbf{C}^k$, where M' is an $(n-k)$-dimensional compact Kähler manifold with quasi-positive Ricci curvature and $\mathbf{C}^k$ is equipped with the flat metric.*

(B) *M' is algebraic, possesses no nonzero holomorphic q-forms for $q \geq 1$, and is holomorphically isometric to a direct product of compact Kähler manifolds $M_1 \times \ldots \times M_s$, where each M_i has quasi-positive Ricci curvature and satisfies $H^2(M_i, \mathbf{Z}) \cong \mathbf{Z}$.*

(C) *There is a flat, compact complex manifold B and a holomorphic, locally isometrically trivial fibration $p: M \to B$ whose fibre is M'.*

(D) *There exists a compact Kähler manifold M^*, a flat complex torus T, and a commutative diagram:*

$$\begin{array}{ccc} M^* & \longrightarrow & T \\ \downarrow & & \downarrow \\ M & \longrightarrow & B \end{array}$$

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