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Power-law subordinacy and singular spectra I. Half-line operators

by

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1. Introduction

In this paper we study one-dimensional Schrödinger operators on the "half-line". We mainly discuss discrete operators on $l^2(\mathbf{Z}^+)$, defined by

$$(H_\theta\psi)(n)=\psi(n+1)+\psi(n-1)+V(n)\psi(n) \tag{1.1}$$

along with a phase boundary condition

$$\psi(0)\cos\theta+\psi(1)\sin\theta=0, \tag{1.2}$$

where $-\frac{1}{2}\pi<\theta<\frac{1}{2}\pi$. The potential $V=\{V(n)\}_{n=1}^{\infty}$ is a sequence of real numbers. While we discuss such discrete operators, our main results (namely, Theorems 1.1 and 1.2 below) are also valid for their continuous analogs of the form $-d^2/dx^2+V(x)$ on $L^2(\mathbf{R}^+)$, as long as the potential $V(x)$ is such that we are in the limit point case (so the operator is essentially self-adjoint). The proofs for the discrete and continuous cases are essentially the same.

While we are mostly interested here in operators of the form (1.1), our core results are valid (and will be proven) for more general tridiagonal operators of the form

$$(H_\theta\psi)(n)=a(n)\psi(n+1)+a(n-1)\psi(n-1)+b(n)\psi(n), \tag{1.3}$$

where the $b(n)$ are real numbers, the $a(n)$ are real and $a(n)\neq 0$ for all n. Moreover, we assume that $\sum_{n=1}^{\infty}|a(n)|^{-1}=\infty$, which is sufficient to ensure that these operators are essentially self-adjoint [2]. The study of an operator of the form (1.3) along with