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Blowup of small data solutions for a class of quasilinear wave equations in two space dimensions, II

by

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Introduction

1. This work is a continuation of our previous work "Blowup of small data solutions for a quasilinear wave equation in two space dimensions" [6]. We consider in both quasilinear wave equations in $\mathbf{R}^{2+1}$,

$$L(u) \equiv \partial_t^2 u - \Delta_x u + \sum_{0 \leqslant i,j,k \leqslant 2} g_{ij}^k \partial_k u \partial_{ij}^2 u = 0, \tag{0.1}$$

where

$$x_0 = t, \quad x = (x_1, x_2), \quad g_{ij}^k = g_{ji}^k.$$

We assume that the Cauchy data are C^∞ and small,

$$u(x,0) = \varepsilon u_1^0 + \varepsilon^2 u_2^0 + \ldots, \quad \partial_t u(x,0) = \varepsilon u_1^1 + \varepsilon^2 u_2^1 + \ldots, \tag{0.2}$$

and supported in a fixed ball of radius M.

We could with minor changes handle as well more general equations of the form

$$\partial_t^2 u - \Delta_x u + \sum g_{ij}(\nabla u) \partial_{ij}^2 u = 0, \tag{0.1$'$}$$

with $g_{ij}(0)=0$, because cubic and higher-order terms play no crucial role in the blowup. We restrict ourselves to (0.1) because previous papers used here have been written in this framework, and also for simplicity.

Following [10], we define

$$g(\omega) = \sum g_{ij}^k \widehat{\omega}_i \widehat{\omega}_j \widehat{\omega}_k, \tag{0.3}$$