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Inverse scattering on asymptotically hyperbolic manifolds

by

MARK S. JOSHI

Natwest Group
London, England, U.K.

and

ANTÔNIO SÁ BARRETO

Purdue University
West Lafayette, IN, U.S.A.

1. Introduction

In this paper, we study scattering for Schrödinger operators on asymptotically hyperbolic manifolds. In particular, we show that the scattering matrix depends meromorphically on the energy $\zeta\in\mathbf{C}$, and for the values of ζ where it is defined, it is a pseudo-differential operator of order $2\operatorname{Re}\zeta-n$ (really complex order $2\zeta-n$), where the dimension of the manifold is $n+1$. We then show that the total symbol of this operator is determined locally by the metric and the potential, and that, except for a countable set of energies, the asymptotics of either the metric or the potential can be recovered from the scattering matrix at one energy. This also allows us to characterize the total symbol of the scattering matrix in the case where the manifold is of product type modulo terms vanishing to infinite order at the boundary.

We remark that the fact that the scattering matrix at energy ζ is a pseudo-differential operator is a known result, see for example §8.4 of [29]. However, the proof in the general case does not seem to be available in the literature. Proofs of several particular cases have been given, see for example [6], [13], [14], [17] and [37], and references given there.

Recall that a compact manifold with boundary $(X,\partial X)$ is asymptotically hyperbolic when it is equipped with a metric of the form

$$g=\frac{H}{x^2},$$

where x is a defining function of ∂X, and H is a smooth Riemannian metric on X, which is smooth and non-degenerate up to ∂X. Furthermore we assume that $|dx|_H=1$ at ∂X. In other words, g can be expressed by

$$g=\frac{dx^2+h(x,y,dx,dy)}{x^2}, \tag{1.1}$$