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The Helgason Fourier transform for compact Riemannian symmetric spaces of rank one

by

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0. Introduction

For Riemannian symmetric spaces (RSS) of noncompact type Helgason [2], [3], [5], found the analog of classical Fourier analysis. This paper concerns the counterpart of Helgason's theory for RSS of compact type. Together with classical Fourier theory these results constitute a unified Fourier analysis on RSS related to, but distinct from the established alternatives of representation theory and the spherical Fourier transform. An advantage of this style of Fourier theory is that the transform kernel has the same kind of simplicity as the functions $e^{ix\cdot y}$ of classical Fourier theory. In particular, the transform kernel employs scalar-valued eigenfunctions of first order differential operators.

On the other hand, the theory given here for the compact RSS involves a severe singularity in part of the transform kernel. One may avoid this singularity by confining consideration to the half of the RSS closest to the origin; call this the *local theory*. The local theory is given in Section 1 for any compact RSS. The rest of this paper is devoted to the global theory for compact RSS of rank one. (It is not clear that a global theory exists for the higher rank compact RSS.)

In broad outline, Helgason's transform comes from the wedding of the spherical Fourier transform with an integral formula for the Poisson kernel. In Helgason's notation ([5], p. 418) this formula is

$$\phi_\lambda(g^{-1}h)=\int_K e^{(-i\lambda+\varrho)(A(kg))}e^{(i\lambda+\varrho)(A(kh))}dk \quad (g,h\in G). \tag{0.1}$$