A constructive proof of the Fefferman-Stein decomposition of BMO $(\mathbf{R}^n)$

by

AKIHITO UCHIYAMA [1]

University of Chicago
Chicago, IL, U.S.A.

1. Introduction

In this note, functions considered are complex-valued unless otherwise explicitly stated. For a function $f(x) \in L^1_{\text{loc}}(\mathbf{R}^n)$, let

$$\|f\|_{\text{BMO}} = \sup |I|^{-1} \int_I |f(x) - f_I| \, dx,$$

where the supremum is taken over all cubes I in $\mathbf{R}^n$, with sides parallel to axis, and where $|I|$ denotes the Lebesgue measure of I and

$$f_I = |I|^{-1} \int_I f(x) \, dx.$$

A function $f(x)$ is said to belong to BMO $(\mathbf{R}^n)$ if $\|f\|_{\text{BMO}} < +\infty$.

Let R_j $(j=1, \ldots, n)$ be the Riesz transforms. That is,

$$R_j f(x) = (-i\xi_j |\xi|^{-1} \hat{f}(\xi))^{\vee}(x),$$

where $i = (-1)^{1/2}$, $\xi = (\xi_1, \ldots, \xi_n)$ and where $\wedge$ and $\vee$ denote the Fourier and the inverse Fourier transforms, respectively. As is well known,

$$R_j f(x) = C_n \, \text{P.V.} \int (x_j - y_j) \, |x-y|^{-n-1} f(y) \, dy$$

for $f(x) \in \bigcup_{1<p<\infty} L^p(\mathbf{R}^n)$. For $f(x) \in L^\infty(\mathbf{R}^n)$, let

[1] This work was supported in part by Science Research Foundation of Japan. (General Research (c) 1980.)