On the regularity of the minima of variational integrals

by

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0. Introduction

The problem of the regularity of functions $u(x)$ minimizing a variational integral

$$F(u;\Omega)=\int_{\Omega} f(x,u,Du)\,dx \tag{0.1}$$

has been one of the main questions since the introduction of direct methods has allowed to pove the existence of minima in suitable classes of generalized functions. It would be impossible to list all the significant contributions since the pioneering work of E. De Giorgi [4]; and we refer to the nowadays classical books by O. A. Ladyženskaya and N. N. Ural'ceva [17] and C. B. Morrey [20].

With extremely few exceptions, all the papers concerned with the regularity problem have as a common starting point the Euler equation of the functional F and therefore require at least some smoothness of the function f and suitable growth conditions for its partial derivatives f_u and f_p.

It goes without saying that the smoothness of f is necessary if one wants to prove the differentiability of the minima; on the other hand, if we look only at the continuity of the solution such assumptions seem superfluous, and it would be preferable to derive it directly from the minimizing property of u.

In addition, it is clear that results obtained from the Euler equation do not distinguish between true minima and simple extremals, and therefore it is sometimes necessary to introduce as hypotheses properties—as for instance the boundedness of the solution—which might hold for minima but are in general false for extremals.

The aim of the paper is to investigate the continuity (in the sense of Hölder) of the minima, directly working with the functional F instead of working with its Euler