THE INHOMOGENEOUS MINIMA OF BINARY QUADRATIC FORMS (IV)

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1. The object of this paper is to show how the ideas of part III of this series may be applied to the problems considered in part I. No results from parts I and II are used, but a knowledge of sections 1 and 2 of part III is essential for an understanding of the method. For convenience of reference, the necessary definitions and theorems are repeated here.

Let $f(x, y) = a x^2 + b x y + c y^2$ be an indefinite binary quadratic form with real coefficients and discriminant $D = b^2 - 4ac > 0$. For any real numbers x_0, y_0 we define $M(f; x_0, y_0)$ to be the lower bound of $|f(x + x_0, y + y_0)|$ taken over all integer sets x, y. The inhomogeneous minimum $M(f)$ of $f(x, y)$ is now defined to be the upper bound of $M(f; x_0, y_0)$ over all sets x_0, y_0. It is convenient to identify pairs of real numbers with points of the Cartesian plane.

As in part III, we approach the problem of evaluating $M(f)$ geometrically, and consider an inhomogeneous lattice $\mathfrak{L}$ in the ξ, η-plane i.e. a set of points with coordinates

$$\begin{aligned} \xi &= \xi_0 + \alpha x + \beta y, \\ \eta &= \eta_0 + \gamma x + \delta y, \end{aligned} \tag{1.1}$$

where $\xi_0, \eta_0, \alpha, \beta, \gamma, \delta$ are real, $\alpha\delta - \beta\gamma \neq 0$, and x, y take all integral values. The determinant of $\mathfrak{L}$ is defined to be

$$\Delta = \Delta(\mathfrak{L}) = |\alpha\delta - \beta\gamma|.$$

If we suppose that $\mathfrak{L}$ has no point on either of the coordinate axes $\xi = 0$, $\eta = 0$, then $\mathfrak{L}$ has at least one divided cell: that is to say, there exist points A, B, C, D of $\mathfrak{L}$, one in each quadrant, such that $ABCD$ is a parallelogram of area Δ.