THE INHOMOGENEOUS MINIMA OF BINARY QUADRATIC FORMS (III)

BY

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1. Introduction

Let $\mathfrak{L}$ be an inhomogeneous lattice of determinant $\Delta = \Delta(\mathfrak{L})$ in the ξ, η-plane, i.e. a set of points given by

$$\begin{aligned} \xi &= \xi_0 + \alpha x + \beta y, \\ \eta &= \eta_0 + \gamma x + \delta y, \end{aligned} \tag{1.1}$$

where $\xi_0, \eta_0, \alpha, \beta, \gamma, \delta$ are real, $\Delta = |\alpha\delta - \beta\gamma| \neq 0$, and x, y take all integral values. In vector notation, $\mathfrak{L}$ is the set of points

$$P = P_0 + xA + yB,$$

where the lattice vectors $A = (\alpha, \gamma)$ and $B = (\beta, \delta)$ are said to generate $\mathfrak{L}$. It is clear that $\mathfrak{L}$ has infinitely many pairs A, B of generators. Corresponding to any such pair and any point P_0 of $\mathfrak{L}$, we call the parallelogram with vertices P_0, $P_0 + A$, $P_0 + B$, $P_0 + A + B$ a *cell* of $\mathfrak{L}$: a parallelogram with vertices at points of $\mathfrak{L}$ is a cell of $\mathfrak{L}$ if and only if it has area Δ.

A cell is said to be *divided* if it has one vertex in each of the four quadrants. Delauney [5] has proved that if $\mathfrak{L}$ has no point on either of the coordinate axes $\xi = 0$, $\eta = 0$, then $\mathfrak{L}$ has at least one divided cell[1]; we outline his proof in § 2. We then develop an algorithm for finding a new divided cell from a given one, thus obtaining in general[2] a chain of divided cells $A_n B_n C_n D_n$ $(-\infty < n < \infty)$. The analytical

[1] This result fills the gap, noted by Cassels [3], in the very simple proof of Minkowski's theorem on the product of two inhomogeneous linear forms given by Sawyer [6].

[2] The condition that the chain does not break off is simply that $\mathfrak{L}$ shall have no lattice-vector parallel to a coordinate axis.