# THE RELATIVELY EXPONENTIAL, LOGARITHMIC AND CIRCULAR FUNCTIONS IN RECURSIVE FUNCTION THEORY

BY

R. L. GOODSTEIN

*of Leicester*

In this paper we develop a theory of relative integration in recursive function theory and apply it to establish the fundamental properties of the relatively exponential, logarithmic and circular functions which are the 'analogues' in the rational space of recursive function theory of the classical functions of the same names. The present account is self contained and may be read without reference to the literature on recursive function theory.[1]

Recursive function theory is a development of a free variable formalisation of arithmetic introduced by Th. Skolem.[2] The elementary formulae of this system are equations between 'terms', and the class of formulae is constructed from the elementary formulae by the operations of the propositional calculus. The terms are the free numeral variables, the sign 0, and the signs for functions. As function signs we have $S(x)$ for the successor function, and signs for functions introduced by recursion. The derivation rules comprise the propositional calculus, the substitution of terms for numeral variables, the schema

$$a = b \to (\mathfrak{a}(a) \to \mathfrak{a}(b)),$$

the induction schema

$$\frac{\mathfrak{a}(o),\ \mathfrak{a}(n) \to \mathfrak{a}(S(n))}{\mathfrak{a}(n)},$$

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[1] A list of publications on recursive function theory is given in the Bibliography of the author's "Constructive Formalism" (Leicester, 1951).

[2] In his paper "Begründung der elementaren Arithmetik durch die rekurrierende Denkweise ...", *Videnskapsselskapets Skrifter* (Kristiania 1923), 2, Vol. I § 7 pp. 3–38.