THE INHOMOGENEOUS MINIMUM OF A TERNARY QUADRATIC FORM

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1. Let $Q(x, y, z)$ be an indefinite ternary quadratic form with real coefficients and determinant $D \neq 0$. Davenport [4] has shown that, given any real numbers x_0, y_0, z_0, there exist x, y, z congruent (modulo 1) to x_0, y_0, z_0 satisfying

$$|Q(x, y, z)| \leq (\tfrac{27}{100}|D|)^{\frac{1}{3}}; \tag{1.1}$$

the equality sign can hold if and only if Q is equivalent (under integral unimodular transformation of the variables) to a multiple of the form

$$Q_1(x, y, z) = x^2 + 5y^2 - z^2 + 5yz + zx.$$

The main weapon used in the proof was a generalization of Minkowski's result on the inhomogeneous minimum of a binary quadratic form, namely:

If $f(x, y)$ is a binary quadratic form with real coefficients and discriminant Δ^2, where $\Delta > 0$, and $\mu > 0$, $\nu > 0$, $\mu\nu \geq \frac{1}{16}$, then, for any real numbers x_0, y_0, there exist $x, y \equiv x_0, y_0 \pmod 1$ satisfying

$$-\nu\Delta \leq f(x, y) \leq \mu\Delta. \tag{1.2}$$

By obtaining an 'isolation' of this inequality when ν is approximately 2μ, Davenport was able to show that the result (1.1) is isolated: that is to say, there exists a positive constant δ such that the inequality

$$|Q(x, y, z)| \leq (1-\delta)(\tfrac{27}{100}|D|)^{\frac{1}{3}} \tag{1.3}$$

can be satisfied whenever Q is not equivalent to a multiple of the special form Q_1.