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Perturbation theory for infinite-dimensional integrable systems on the line. A case study.

by

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In memory of Jürgen Moser

Contents

1. Introduction

In this paper we consider perturbations

$$\begin{gathered} iq_t+q_{xx}-2|q|^2q-\varepsilon|q|^lq=0, \\ q(x,t=0)=q_0(x)\to 0 \quad \text{as } |x|\to\infty \end{gathered} \tag{1.1}$$

of the defocusing nonlinear Schrödinger (NLS) equation

$$\begin{gathered} iq_t+q_{xx}-2|q|^2q=0, \\ q(x,t=0)=q_0(x)\to 0 \quad \text{as } |x|\to\infty. \end{gathered} \tag{1.2}$$

Here $\varepsilon>0$ and $l>2$. The particular form of the perturbation $\varepsilon|q|^lq$ in (1.1) is not special, and it will be clear to the reader that the analysis goes through for any perturbation of the form $\varepsilon\Lambda'(|q|^2)q$, as long as $\Lambda: \mathbf{R}_+\to\mathbf{R}_+$ is sufficiently smooth, $\Lambda'(s)\geqslant 0$ and $\Lambda(s)$

A more detailed, extended version of this paper is posted on `http://www.ml.kva.se/publications/acta/webarticles/deift`. Throughout this paper we refer to the web version as [DZW].