THE MAXIMUM MODULUS AND VALENCY OF FUNCTIONS MEROMORPHIC IN THE UNIT CIRCLE.

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Chapter III.

Converse Theorems.

1) The aim of this chapter is to prove converse theorems to the results of Chapter II. We remind the reader of the fundamental problem, which is to investigate the rate of growth of the maximum modulus of a function $f(z)$, meromorphic in $|z| < 1$, which takes none of a set E of complex values more than $p(\varrho)$ times in $|z| < \varrho$, $0 < \varrho < 1$. In this chapter we shall construct examples to show that all the results we have proved give the correct order of magnitude for $\log M[\varrho, f]$ when

$$p(\varrho) \equiv (1-\varrho)^{-a}, \quad 0 \leq a < \infty. \tag{1.1}$$

The functions $f(z)$ which we construct will be regular, nonzero so that $f(z) = f_*(z)$.

We remind the reader of the four separate problems we considered in the latter half of Chapter II as stated in paragraph 19 of that chapter.

(i) *What results hold if E contains the whole w plane?*

(ii) *What sets E have the same effect as the whole plane for a given function $p(\varrho)$?*

(iii) *What results hold if we merely assume that E contains some arbitrarily large values?*

(iv) *What results hold if we assume merely that E contains ∞ and at least two finite values or a bounded set?*

The positive theorem in case (i) was proved in Theorem VII, Corollary, of Chapter II. In the case of (1.1) above this result yields