Quasiconformal homeomorphisms and dynamics II: Structural stability implies hyperbolicity for Kleinian groups

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Introduction

Let $\Gamma_0 \subset \{z \mapsto (az+b)/(cz+d);\ a, b, c, d, \text{ in } \mathbf{C},\ ad-bc=1\} = PSl(2, \mathbf{C})$ be a finitely generated *non-solvable* subgroup of the orientation-preserving conformal transformations of the Riemann sphere $\bar{\mathbf{C}}$. Say that Γ_0 is *structurally stable* if all sufficiently near representations into $PSl(2, \mathbf{C})$ are injective—there are no new relations. Say that Γ_0 is *non-rigid* if there are arbitrarily close representations which are not conjugate in $PSl(2, \mathbf{C})$. Otherwise Γ_0 is *rigid*. If Γ_0 is discrete and the fundamental domain in hyperbolic 3-space is finite sided one says Γ_0 is *geometrically finite*. An element γ in $PSl(2, \mathbf{C})$ is *parabolic* iff trace $\gamma = a+d$ is ± 2. For torsion free groups we have,

THEOREM A. *A structurally stable $\Gamma_0 \subset PSl(2, \mathbf{C})$ is either rigid or it is a discrete geometrically finite group with no non-trivial parabolic elements.*

The condition *geometrically finite without parabolics* is equivalent (§ 9) in the context of discrete subgroups of $PSl(2, \mathbf{C})$ to an expanding property for the action of Γ_0