Two theorems of N. Wiener for solutions of quasilinear elliptic equations

by

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1. Introduction

Relatively little is known about boundary behavior of solutions of quasilinear elliptic partial differential equations as compared to that of harmonic functions. In this paper two results, which in the harmonic case are due to N. Wiener, are generalized to a non-linear situation. Suppose that G is a bounded domain in $\mathbf{R}^n$. We consider functions $u: G \to \mathbf{R}$ which are free extremals of the variational integral

$$\int F(x, \nabla u(x))\, dm(x)$$

in the conformally invariant or borderline case $F(x,h) \approx |h|^n$. For the precise assumptions on the kernel $F: G \times \mathbf{R}^n \to \mathbf{R}$ see Section 2. Equivalently, the F-extremality of u means that u is a weak solution of the corresponding Euler equation

$$\nabla \cdot \nabla_h F(x, \nabla u(x)) = 0 \tag{1.1}$$

with measurable coefficients. Solutions u, *F-extremals*, of the equation (1.1) form a similar basis for a non-linear potential theory as harmonic functions do for the classical potential theory. Especially, the Perron-Wiener-Brelot method can be applied, see Section 2.10. For each bounded function $f: \partial G \to R$ there exist two F-extremals, the *upper Perron solution* $\bar{H}_f$ and the *lower Perron solution* $\underline{H}_f$ with "boundary values" f in G. These functions are defined via sub- and super-solutions as in the classical harmonic case. In 1970, W. Mazja [M] proved, although his formulation was slightly less general, that if f is continuous and if the Wiener condition