The Corona theorem for Denjoy domains

by

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§ 1. Introduction

Denote by $H^\infty(\mathcal{D})$ the space of bounded analytic functions on a plane domain $\mathcal{D}$ and give functions in $H^\infty(\mathcal{D})$ the supremum norm

$$\|f\| = \sup_{z\in\mathcal{D}} |f(z)|.$$

A Denjoy domain is a connected open subset Ω of the extended complex plane $\mathbf{C}^*$ such that the complement $E=\mathbf{C}^*\setminus\Omega$ is a subset of the real axis $\mathbf{R}$.

THEOREM. *If Ω is any Denjoy domain and if $f_1, \ldots, f_N \in H^\infty(\Omega)$ satisfy*

$$0<\eta\leqslant \max_j |f_j(z)| \leqslant 1 \tag{1.1}$$

for all $z\in\Omega$, then there exist $g_1, \ldots, g_N \in H^\infty(\Omega)$ such that

$$\sum f_j(z)\, g_j(z) = 1, \quad z\in\Omega. \tag{1.2}$$

Such a theorem is called a corona theorem (had the theorem been false for Ω the unit disc, there would have been a set of maximal ideals suggestive of the sun's corona), and the g_j are called corona solutions. It follows from the methods in Gamelin [6] that the theorem is equivalent to itself plus the further conclusion

$$\|g_j\| \leqslant C(N, \eta),$$

(1) Partially supported by NSF grants MCS-8002955 and MCS-8311513.