TRANSFORMATIONS OF CERTAIN HYPERGEOMETRIC FUNCTIONS OF THREE VARIABLES

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1. There are several methods for obtaining transformations of hypergeometric functions of three variables. The first and simplest is by writing the triple series defining a given hypergeometric function as an infinite sum of the hypergeometric functions of two variables; the known transformation theory can then be applied to each term to obtain new transformations.

The second method consists in transforming the system of partial differential equations satisfied by these hypergeometric functions. This method is rather tedious in practice and not very useful for discovering new transformations.

The third method is obtained by transformation of integrals representing these functions. The object of this paper is to apply the third method to obtain some new transformations of such functions. The first two methods have been illustrated by me [4]. The success of the present method, as is obvious, lies in the method of substitution in the integral representations known for our functions and as such it becomes less useful in the cases where the integrals are such that substitutions are not very elegant.

2. Following the notation of [4] the hypergeometric functions of three variables are defined as

$$(2.1)\quad F_E(\alpha_1, \alpha_1, \alpha_1, \beta_1, \beta_2, \beta_2;\ \gamma_1, \gamma_2, \gamma_3;\ x, y, z) = \sum \frac{(\alpha_1, m+n+p)\,(\beta_1, m)\,(\beta_2, n+p)}{(1, m)\,(1, n)\,(1, p)\,(\gamma_1, m)\,(\gamma_2, n)\,(\gamma_3, p)}\, x^m\, y^n\, z^p,$$

$$(2.2)\quad F_F(\alpha_1, \alpha_1, \alpha_1, \beta_1, \beta_2, \beta_1;\ \gamma_1, \gamma_2, \gamma_2;\ x, y, z) = \sum \frac{(\alpha_1, m+n+p)\,(\beta_1, m+p)\,(\beta_2, n)}{(1, m)\,(1, n)\,(1, p)\,(\gamma_1, m)\,(\gamma_2, n+p)}\, x^m\, y^n\, z^p,$$