ON THE DIFFERENTIAL EQUATIONS OF HILL IN THE THEORY OF THE MOTION OF THE MOON

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1. G. W. Hill's equations define the movement of a body with infinitely small mass, attracted by Newton's law of gravitation towards two bodies moving in circles by that same law. Of these two bodies the mass of one is negligible in comparison with that of the other, and the distance of the body with infinitely small mass from the smaller of the two other bodies is assumed to be negligible in comparison with the distance between these. The whole movement takes place in a plane and is referred to the uniformly rotating axes. It is, thus, a degenerate case of the problem of three bodies.

Hill employed rectangular coordinates with the time as independent variable. Let p and q be the coordinates, t the time, then Hill's equations are[1]

$$\left\{\begin{aligned} \frac{d^2 p}{d t^2} - 2\frac{d q}{d t} &= 3p - \frac{p}{r^3} \\ \frac{d^2 q}{d t^2} + 2\frac{d p}{d t} &= -\frac{q}{r^3} \end{aligned}\right. \tag{1}$$

with Jacobi's integral

$$\left(\frac{d p}{d t}\right)^2 + \left(\frac{d q}{d t}\right)^2 = 3p^2 + \frac{2}{r} - C. \tag{2}$$

It has been shown[2] that, introducing polar coordinates $p = r\cos l$, $q = r \sin l$, putting at the same time

[1] See, for instance, H. C. PLUMMER: *An introductory treatise on dynamical astronomy* (1918), 265–6.

[2] J. F. STEFFENSEN: Les orbites périodiques dans le problème de Hill. *Académie royale de Danemark, Bulletin* 1909 n$^{\text{o}}$ 3, 320–3.

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