FINITE DIMENSIONAL CONVOLUTION ALGEBRAS

BY

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Introduction

The notion of convolution (Ger. *Faltung*, Fr. *produit de composition*) is a venerable one in mathematical analysis. The convolution

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \varphi(\alpha)\frac{\sin\,(n+\frac{1}{2})\,(\alpha-x)}{2\,\sin\,\frac{1}{2}\,(\alpha-x)}\,d\alpha$$

is found in Dirichlet's original memoir [12] on Fourier series, and similar convolutions are extensively used in the classical literature on Fourier series and integrals (see for example Riemann [34]. Weierstrass's original proof [47] of the celebrated approximation theorem bearing his name utilizes a certain convolution. Fractional integration and differentiation are defined by means of convolutions (see for example Zygmund [51], pp. 222–225). A perusal of any adequate textbook on Fourier series or integrals will show the important place occupied by the notion of convolution in the field of harmonic analysis. The Hilbert transform is of course a convolution. The classical theory of this transform has recently been extended by Zygmund and Calderon [52].

More recently, it has been recognized that measures and certain classes of abstract linear functionals can be convolved. For functions of finite variation on $(-\infty, +\infty)$, for example, see Bochner [5], pp. 64–74, and, from another point of view, Beurling [4] and Gel'fand [15]. The notion is discussed and utilized *in extenso* by Jessen and Wintner [22]. L. Schwartz has studied convolutions for the class of linear functionals

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