ON SYMBOLIC CALCULUS OF TWO VARIABLES

BY

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in Calcutta

1. The notation

$$\phi(p, q) \leqslant h(x, y)$$

between the image and the object in the symbolic calculus of two variables is used to represent the convergent double integral

$$\phi(p, q) = pq \int_0^\infty \int_0^\infty e^{-px-qy} h(x, y)\, dx\, dy. \quad R(p) > 0, \ R(q) > 0.$$

The object of the present paper is to derive a theorem in the symbolic calculus of two variables, starting from a chain of relations in one variable, and to show some applications of the theorem.

2. **Theorem.** If $f(p) \subset x^{\nu-1} h(x)$ and $p^{\mu-\lambda} h\left(\frac{1}{p^\mu}\right) \subset \sigma(x)$, then

$$y^{\nu-\frac{\lambda}{\mu}} \sigma(x y^{\frac{1}{\mu}}) \geqslant p^{\mu\nu-\lambda} f(p^\mu q),$$

valid when $\mu > 0$, $R(\lambda) > -1$, $R(\nu) > -1$.

Proof. Let $J_\lambda^\mu(x)$ represent Wright's generalised Bessel function [1] defined by

$$J_\lambda^\mu(x) = \sum_{r=0}^\infty \frac{(-x)^r}{r!\,\Gamma(1+\lambda+\mu r)}, \quad \mu > 0, \ R(\lambda) > -1.$$

Then

$$x^\lambda y^\nu J_\lambda^\mu(x^\mu y) = \sum_{r=0}^\infty \frac{(-1)^r x^{\lambda+\mu r} y^{\nu+r}}{r!\,\Gamma(1+\lambda+\mu r)}$$

$$\geqslant \frac{1}{p^\lambda q^\nu} \sum_{r=0}^\infty \frac{\Gamma(\nu+r+1)}{r!} \left(-\frac{1}{p^\mu q}\right)^r$$

$$= \Gamma(\nu+1) \cdot \frac{p^{\mu(\nu+1)-\lambda} \cdot q}{(1+p^\mu q)^{\nu+1}}, \quad R(\nu) > -1. \tag{2.1}$$