ON THE CONNECTEDNESS OF DEGENERACY LOCI AND SPECIAL DIVISORS

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Introduction

Let C be a smooth complex projective curve of genus g, and let J be the Jacobian of C. Upon choosing a base-point in C, J may be identified with the set of linear equivalence classes of divisors of degree d on C. Denote by W_d^r the algebraic subvariety of J parametrizing divisors which move in a linear system of dimension at least r. A fundamental theorem of Kempf [9] and Kleiman and Laksov [11, 12] asserts that these loci are non-empty when their expected dimension

$$\varrho = g-(r+1)(g-d+r)$$

is non-negative. We complement this existence theorem with two results on the global structure of W_d^r when $\varrho > 0$. First of all, for an arbitrary curve C, we prove

THEOREM I. *If $\varrho > 0$, then W_d^r is connected.*

When C is generic (in the sense of moduli), deep results about the local geometry of W_d^r have been obtained by Griffiths and Harris [5] and by Gieseker [4]. Combining these with Theorem I, we deduce the

COROLLARY. *For a generic curve C, W_d^r is irreducible when $\varrho > 0$.*

By a standard construction, W_d^r may be realized as the locus where a certain homomorphism of vector bundles on J drops rank. Theorem I then becomes a simple consequence of a general result—of independent interest—on the connectivity of such degeneracy loci.

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