ON THE NIELSEN–THURSTON–BERS TYPE OF SOME SELF-MAPS OF RIEMANN SURFACES(1)

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§ 1. Introduction

Let S be a surface of non-excluded (see § 2) finite type, and set $\dot{S}=S\setminus\{x_0\}$ for some $x_0\in S$. Consider the very simplest self-maps of $\dot{S}$: the self-maps that are homotopic to the identity on S (in particular such maps must fix x). When is such a map parabolic, hyperbolic, or pseudohyperbolic (see § 4 for definition) in the sense of Bers [9]? When is such a map reducible (see § 2) in the sense of Thurston [36]? We give a complete answer to this question, and as a consequence obtain two interesting facts:

(I) There exist irreducible self-mappings on Riemann surfaces of every non-excluded type $(p, n)\neq(0, 3)$; that is, as long as $3p+n>3$.

(II) The Teichmüller (=Kobayashi) metric on the fibers of the Bers fiber spaces is not (a multiple of) the Poincaré metric on the fibers, unless the Bers fiber space is one dimensional.

The more exact formulation of our first important result is summarized in

THEOREM 2′. *Let S be an oriented surface of non-excluded finite type. Let $x_0\in S$ and set $\dot{S}=S\setminus\{x_0\}$. Let w be a self-map of S with $w(x_0)=x_0$ and w isotopic to the identity on S. Let J be an isotopy of w to the identity:*

$$J\colon [0, 1]\times S\to S,$$

$$J(0, x)=w(x),$$

and

$$J(1, x)=x.$$

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