ANALYTIC RAMIFICATIONS AND FLAT COUPLES OF LOCAL RINGS

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Introduction

In a paper of 1935 Akizuki constructed an analytically ramified (Noetherian) local domain of Krull dimension one ([1], Section 3). We shall present another, similar construction. It effects a transformation

$$(R_0, R_1) \to \{R, \mathfrak{p}\}$$

where on the left stands an arbitrary equidimensional flat couple of local rings and on the right a local ring together with a prime ideal (of coheight one) whose analytic ramification reflects the structure of the couple to the left. More precisely, the completion $\hat{R}$ of R contains just one prime ideal $\mathfrak{p}^*$ contracting to $\mathfrak{p}$, and the couple $(R_{\mathfrak{p}}, \hat{R}_{\mathfrak{p}^*})$ mirrors the structure of (R_0, R_1) inasmuch as there exists a commutative diagram

$$\begin{array}{ccc} R_0 & \longrightarrow & R_{\mathfrak{p}} \\ \downarrow & & \downarrow \\ R_1 & \longrightarrow & \hat{R}_{\mathfrak{p}^*} \end{array}$$

with unramified flat ring injections as horizontal maps. (See below for definitions.)

Two conclusions can be drawn from this construction (cf. further [11]). One is simply that there are plenty of analytic ramifications. The prime information in this respect is obtained already by taking for R_0 a field K and for R_1 a ring A of the form $K[Z_1, ..., Z_n]/I$ with I primary for $(Z_1, ..., Z_n)$. Then $\mathfrak{p}$ must be equal to (0) so that R becomes a one-dimensional local domain with the property that $\hat{R}_{\mathfrak{p}^*}$, the formal fiber of its zero ideal, is an unramified flat extension of A. Actually $\hat{R}_{\mathfrak{p}^*} \simeq A \otimes_K K((x))$ (where $K((x)) = K[[x]][x^{-1}]$), as is easily derived from the explicit formulas $\hat{R} = \hat{R}_1[[x]]$, $\mathfrak{p}^* = \mathfrak{m}_1 \hat{R}$ (cf. below).