

SUBANALYTIC SETS IN THE CALCULUS OF VARIATION

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0. Introduction

In this paper we shall study a type of analytic extreme value problems depending on parameters. More precisely, the purpose is to study the singularities of the extreme value as a function of these parameters. The key to all that follows is the concept of subanalytic functions. These are functions whose graphs are subanalytic in the sense of Hironaka [5]. In fact, in Section 3.2 of this paper, we shall see that under rather general circumstances, extreme value functions are subanalytic, hence their singularities are amenable to the rather detailed analysis in Chapter 2. As a by-product, we obtain some results in analytic geometry, for example that the singular set of a subanalytic set is subanalytic.

The main motivation for this work however, depends on the fact that the abstract machine can be applied in different areas of mathematics to give interesting results. In