SYMBOLIC DYNAMICS FOR GEODESIC FLOWS

BY

CAROLINE SERIES

University of Warwick, Coventry, England

Introduction

By the classical result of Hopf [12], the geodesic flow on a surface of constant negative curvature and finite area is ergodic. In the case of a compact surface the flow has subsequently been shown to be Anosov [2], K [17], and Bernoulli [15]. By the work of Bowen and Ruelle [5] any Anosov flow on a compact manifold can be represented as a special flow over a Markov shift of finite type, with a Hölder continuous height function. Ratner [16] showed that any such special flow which is K is also Bernoulli.

In this paper we make an explicit geometrical construction of a symbolic dynamics for the geodesic flow on a surface of constant negative curvature and finite area. The construction involves the geometry of the surface and the structure of its fundamental group. The geodesic flow is shown to be a quotient of a special flow over a Markov shift, by a continuous map which is one—one except on a set of the first category. For a compact surface the height function is Hölder.

The states for the Markov shift are generators of the fundamental group Γ, and the admissible sequences are determined by the relations among the generators. If we lift the surface to its universal covering space the unit disc D, then admissible sequences correspond to geodesics in D which pass close to a fixed central fundamental region for Γ, in a sense made precise in § 3. The height function h corresponds to the time a geodesic takes to cross R, with a suitable convention if the geodesic is close to R but does not cut R.

The idea of our construction comes from three different sources. In [3] Artin obtained a representation of geodesics in the Poincaré upper half plane H (these geodesics are of course semi-circles centred on and orthogonal to the real axis) as doubly infinite sequences of positive integers, by juxtaposing the continued fraction expansions of their endpoints; two geodesics are then conjugate under the action of GL $(2, \mathbf{Z})$ on H if and only if the corresponding sequences are shift equivalent.

The second source is Hedlund's paper [11]. In [14] Nielsen gave a symbolic representa-