THE STRUCTURE OF FINITE DIMENSIONAL BANACH SPACES WITH THE 3.2. INTERSECTION PROPERTY

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1. Introduction

Let X be a Banach space over the real numbers. Let n and k be integers with $2 \leqslant k < n$. We say that X has the $n.k.$ intersection property ($n.k.$I.P.) if the following holds:

Any n balls in X intersect provided any k of them intersect.

In [2], O. Hanner characterized finite dimensional spaces with the 3.2.I.P. by the facial structure of their unit hall. He also proved that this property is preserved under l_1- and l_∞-summands, i.e. direct sums $X \oplus Y$ with the l_1-norm $\|x\| + \|y\|$ or the l_∞-norm $\max(\|x\|, \|y\|)$. We shall prove the converse of this result. Any finite dimensional Banach space X with the 3.2.I.P. is obtained from the real line by repeated l_1- and l_∞-summands. Hanner proved this for dimension at most 5.

In sections 2 to 4 we gradually introduce the concepts and theorems that we need. To become familiar with the techniques involved, we have included the proof of some of the results. In sections 5 and 6 we prove some technical lemmas and characterize the parallel-faces and split-faces among the faces of the unit balls of Banach spaces with the 3.2.I.P. These results are used in the proof of the main result in section 7.

Banach spaces are denoted X, Y, and Z. The closed ball in X with center x and radius r is denoted $B(x, r)$, but for the unit ball we write $X_1 = B(0, 1)$. The dual space of X is written X^*. The convex hull of a set S is written conv (S) and the set of extreme points

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