ON THE PROJECTION OF A PLANE SET OF FINITE LINEAR MEASURE *

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Introduction

For any set X we denote by $\Lambda(X)$ the linear measure (1) of X and by $\Lambda(X_\theta)$ the linear measure of the projection of X onto a line perpendicular to the direction θ. We write $\mu(X)$ for the greatest lower bound of $\Lambda(X_\theta)$ taken over all directions θ. We shall consider three classes of planar sets, namely measurable sets, connected sets, and arcs. For each class we shall find the upper bound of the ratio $\mu(X)/\Lambda(X)$.

For the class of measurable sets the result is connected with the properties of regular and irregular sets and is a consequence of the properties of these sets established by Besicovitch. For the class of connected sets and for the class of arcs $\mu(X)$ is the minimal width of the convex cover of X or its convex hull as it is sometimes called. The problem of the relationship between this function and $\Lambda(X)$ is one between a set and its convex cover. There are of course a large number of such properties and a further result of this type is given in Section 5.

An interesting feature of this problem is the difficulty of determining completely the class of extremal figures. For the class of measurable sets the upper bound of $\mu(X)/\Lambda(X)$ is never attained, but we give examples to show that the upper bound which we establish is in fact the least upper bound. On the other hand both the upper bounds for the class of connected sets and for the class of arcs are attained; in the first class by a set composed of three equal segments equally inclined to one another and in the second case by an arc

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* *Editor's note.*—This paper was received on January 4, 1957. Without our knowledge it has appeared during 1957 as part of the book *Problems in Euclidean Space*, London 1957, by the same author.

(1) Hausdorff one-dimensional measure. See [1], where it is referred to as Carathéodory measure.