A CHARACTERIZATION OF DOUGLAS SUBALGEBRAS

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1. Introduction

Let L^∞ be the complex Banach algebra of bounded Lebesgue measurable functions on the unit circle ∂D in the complex plane. The norm in L^∞ is the essential supremum over ∂D. Via radial limits, the algebra H^∞ of bounded analytic functions on the unit disc D forms a closed subalgebra of L^∞. This paper studies the closed subalgebras B of L^∞ properly containing H^∞. For such an algebra B we let B_I denote the closed algebra generated by H^∞ and the complex conjugates of those inner functions which are invertible in the algebra B. (An inner function is an H^∞ function unimodular on ∂D). It is clear that $B_I \subset B$. R. G. Douglas [4] has conjectured that $B = B_I$ for all B, and consequently algebras of the form B_I are called Douglas algebras.

A discussion of the Douglas problem and a survey of related work can be found in [11]. In particular, it is noted in [11] that the maximal ideal space $\mathcal{M}(B)$ of B can be identified with a closed subset of $\mathcal{M}(H^\infty)$, and when B is a Douglas algebra, $\mathcal{M}(B)$ completely determines B. This means that if the Douglas question has an affirmative answer then distinct algebras B has distinct maximal ideal spaces. That the latter assertion is true when one of the algebras is a Douglas algebra is the main result of this paper. We prove that if B and B_1 are closed subalgebras of L^∞ containing H^∞, if $\mathcal{M}(B) = \mathcal{M}(B_1)$ and if B is a Douglas algebra, then $B = B_1$. Using this theorem, D. E. Marshall [9] has answered the Douglas question affirmatively.

Using functions of bounded mean oscillation, D. Sarason [13] had proved the theorem above in the special case when B is generated by H^∞ and the space of continuous functions on ∂D. By similar means, S. Axler [1], T. Weight [15] and the author [3] had verified the theorem for some other specific Douglas algebras.

Section 2 contains some preliminary definitions and lemmas. The more technical aspects of the proof are in section 3 and the main theorem is proved in section 4. Some

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