ON THE REPRESENTATION OF A NUMBER AS THE SUM OF TWO SQUARES AND A PRIME

BY

C. HOOLEY

Corpus Christi College, Cambridge

1. Introduction

In their celebrated paper "Some problems of partitio numerorum: III" [6] Hardy and Littlewood state an asymptotic formula, suggested by a purely formal application of their circle method, for the number of representations of a number n as the sum of two squares and a prime number. The truth of this formula would imply that every sufficiently large number is the sum of two squares and a prime. No proof, even on the extended Riemann hypothesis (which we hereafter refer to as Hypothesis R), has hitherto been found. However, in another paper [7] they suggest that on Hypothesis R it should be possible to prove that *almost all* numbers can be so represented. This proof was effected by Miss Stanley [11], as were proofs (also on Hypothesis R) of asymptotic formulae for the number of representations of a number as sums of *greater* numbers of squares and primes. The dependence of her results on the unproved hypothesis was gradually removed by later writers, in particular by Chowla [2], Walfisz [13], Estermann [4] and Halberstam [5].

It is the purpose of this paper to shew that the original formula of Hardy and Littlewood is true on Hypothesis R. Our method depends on the fact that, as is easily seen, the number of representations of n in the required form is equal to the sum

$$\sum_{p<n} r(n-p),$$

where $r(\nu)$ denotes the number of representations of ν as the sum of two integral squares. On noting that $r(\nu)$ may be expressed as a sum over the divisors of ν, we see that our problem is related in character to the problem of determining the asymptotic behaviour of the sum

$$\sum_{0<p+a\leq x} d(p+a),$$